ABOUT A NONLINEAR GENERALIZATION OF THE CONTRACTION MAPPING

Vantsyan A.A., vantsyan@mechins.sci.am

Gevorgyan H.A., hrgevorkian@mail.ru

Abstract. At first, here a generalize method of iteration is proposed, presented as a combination of the classical iteration and proportional division methods, on which the conditions of Bolzano-Cauchy theorem are satisfied. An evidence of the proposed algorithm convergence is brought. Originally, a generalize contraction mapping is considered as a function from one variable for which a theorem about iteration convergence and its evidence are brought. Secondly, in this article a variant of the generalize method of iteration – a nonlinear-generalize method of iteration – is developed. A new geometrical interpretation of the convergence of the generalize method of iteration is brought: three cases of step, spiral and hyper-step iterations are estimated and their convergence sub-regions are considered. An explicit formula of nonlinear-generalize contraction mapping operator as a function from one real variable is obtained; a formulation of the nonlinear-generalize contraction mapping as a function from complex and some real variables are also explicitly exposed. As a result, an aggregate method of iteration is formulated. On examples of some transcendent equations systems solution an advantage of this method compared to such known methods as the classical iteration method and the Newton's method is proved.

Keywords: generalize and nonlinear methods of iteration, contraction mapping, nonlinear-generalize contraction mapping, contractible original, convergence, fixed point, aggregate method of iteration, convergence, transcendent equations systems.

1. Introduction. In the publications [1 – 5] the full justification of a theoretical correctness and applied efficiency of the *generalize method of iteration* (GMI), concerned to nonlinear algebraic and transcendent equations solutions, were given. Moreover, as was showed in the works [3 - 5], the GMI can be qualified as more efficient numerical method of nonlinear algebraic, transcendent and differential equations solutions than such methods as the methods of tangent (Newton method), of secant and the combined method.

In the present article an extension of traditional ideas about the contraction mapping [6], as the fundamental principle of algebraic, transcendent and differential equations solution by successive approximations principle, is planned. Here is proposed a nonlinear generalization of the operator of contraction mapping that serve a theoretical basis for a new correct and complete methodology destined to any complexity nonlinear equations and systems numerical solution.

2. Original idea of generalization of the method of iteration. As an illustration of the original idea for the generalization of the classical method of iteration, first of all let as give the algorithm of the numerical solution of nonlinear algebraic equations, initially presented in [1], later named *modernize method of iteration* (MMI) [2]:

1. Specification of an initial approximation $x_0 \in [a,b]$;
2. Computation of $x_1 = \phi(x_0)$;
3. Computation of $x_2 = \frac{x_0 + x_1}{2}$;
4. Calculation of $x_3 = \phi(x_2)$;
5. Computation of $x_4 = \frac{x_2 + x_3}{2}$ etc.

For the proof of the process convergence the known estimate [7] is used:

1. $|x_1 - \xi| = |\phi(x_0) - \phi(\xi)| \le q|x_0 - \xi|, \quad |q| < 1;$

2. $$|x_2 - \xi| = \left|\frac{x_0 + x_1}{2} - \xi\right| = \frac{1}{2}|(x_0 - \xi) + (x_1 - \xi)| \le$$
$$\le \frac{1}{2}|x_0 - \xi| + \frac{1}{2}|x_1 - \xi| \le \frac{1}{2}|x_0 - \xi| + \frac{1}{2}q|x_0 - \xi| =$$
$$= \frac{1}{2}(q+1)|x_0 - \xi|,$$

after which it’s not difficult to find the estimate for the n – th iteration

$$|x_n - \xi| \le \frac{1}{2^{n/2}}(q+1)^{n/2}|x_0 - \xi|, \quad \text{если} \quad n = 2k, \quad k = 0,1,2,...;$$

$$|x_n - \xi| \le \frac{1}{2^{(n-1)/2}}(q+1)^{(n-1)/2}|x_0 - \xi|, \quad \text{если} \quad n = 2k+1, \quad k = 0,1,2,....$$

For a sufficiently large n, the last two estimates can be replaced by the expression

$$|x_n - \xi| \le \lim_{n\to\infty}\left[\frac{1}{2^{n/2-1/2}}(q+1)^{n/2-1/2}\right]|x_0 - \xi| = \lim_{n\to\infty}\left(\frac{q+1}{2}\right)^{n/2}|x_0 - \xi|.$$

By virtue $|q|<1$ for the classical iteration, it follows that $\left|\frac{q+1}{2}\right|<1$, i.e. $-3<q<1$, which confirms the convergence of the iteration process.

3. Nonlinear-generalize contraction mapping as a function from one variable. A view to illustrate a new generalization of traditional contraction mapping operator let us consider its primarily in the form of simple function from one variable.

As it's known [6], an arbitrary function $\varphi=\varphi(x)$, satisfying *the Lipchitz condition:* $\varphi(x)=x$, if $|\varphi(x_1)-\varphi(x_2)|\le q|x_1-x_2|$ for $0<q<1$, is called *contraction mapping*.

Introducing the function as a *generalize contraction mapping*:

$$\Phi(x)=\frac{\psi(x)+(\lambda-1)x}{\lambda}, \tag{3.1}$$

one can prove that practically for any function $\psi=\psi(x)$, it is a classical contraction mapping in the complete metric space $\boldsymbol{R}$, i.e. $|\Phi'(x)|\le q<1$.

Theorem. Let the function $\varphi(x)$ is continuous and differentiable in the interval $[a,b]$, moreover for all its values one has $\varphi(x)\in[a,b]$. Then if there exists a proper fraction q, such that

$$\frac{|\lambda-1+\varphi'(x)|}{\lambda}\le q<1 \tag{3.2}$$

for $a<x<b$ and $\lambda\in R$, then the generalize process of the iteration

$$x_n=\frac{\varphi(x_{n-1})+(\lambda-1)x_{n-1}}{\lambda},\quad n=1,2,... \tag{3.3}$$

converges, independent from the initial value $x_0\in[a,b]$, to the limiting value

$$\xi=\lim_{n\to\infty}x_n, \tag{3.4}$$

which is a unique root for the equation

$$f(x)=0, \quad \text{or} \quad \varphi(x)=x, \tag{3.5}$$

in the interval $[a,b]$, where the condition of the Bolzano-Cauchy theorem on the existence of an isolated root (3.4) of the equation (3.5) is satisfied: $f(a)\cdot f(b)<0$.

Proof. Let us consider two successive approximations:

$$x_n=\frac{\varphi(x_{n-1})+(\lambda-1)x_{n-1}}{\lambda} \quad \text{and} \quad x_{n+1}=\frac{\varphi(x_n)+(\lambda-1)x_n}{\lambda},$$

from which it follows that

$$\varphi(x_n)-\varphi(x_{n-1})+(\lambda-1)(x_n-x_{n-1})=\lambda(x_{n+1}-x_n).$$

Applying the Lagrange's theorem, we get

$$(x_n-x_{n-1})\varphi'(\bar{x}_n)+(\lambda-1)(x_n-x_{n-1})=\lambda(x_{n+1}-x_n), \quad \bar{x}_n\in(x_{n-1},x_n);$$

i.e.

$$|x_{n+1}-x_n|=\left|\frac{\lambda-1+\varphi'(\bar{x}_n)}{\lambda}\right||x_n-x_{n-1}|,$$

or, by taking into account the condition (3.2),

$$|x_{n+1}-x_n|\le q|x_n-x_{n-1}|. \tag{3.6}$$

From here giving the values $n=1,2,\ldots,$ successively one can obtain

$$|x_2-x_1|\le q|x_1-x_0|,$$
$$|x_3-x_2|\le q|x_2-x_1|\le q^2|x_1-x_0|,$$

$$|x_{n+1}-x_n|\le q^n|x_1-x_0|.$$

Consider the series

$$x_0+(x_1-x_0)+(x_2-x_1)+\ldots+(x_n-x_{n-1})+\ldots, \tag{3.7}$$

for which the successive approximations x_n are the $(n+1)$-th partial sums, i.e. $x_n=S_{n+1}$.

By virtue inequality (3.6) the terms of series (3.7) by the absolute value are less than the corresponding terms of the geometrical progression with the denominator $q<1$. Therefore the series (3.7) converges absolutely. And from here it follows that for a continuous function $\varphi(x)$ there is the limit:

$$\lim_{n\to\infty}S_{n+1}=\lim_{n\to\infty}x_n=\frac{(\lambda-1)\lim\limits_{n\to\infty}x_{n-1}+\varphi\left(\lim\limits_{n\to\infty}x_n\right)}{\lambda}=\frac{(\lambda-1)\xi+\varphi(\xi)}{\lambda}=\xi,$$

which unlimitedly is equivalent to the results $\varphi(\xi) = \xi$, from which it follows that ξ is a root of the equation (3.5), what was required to prove.

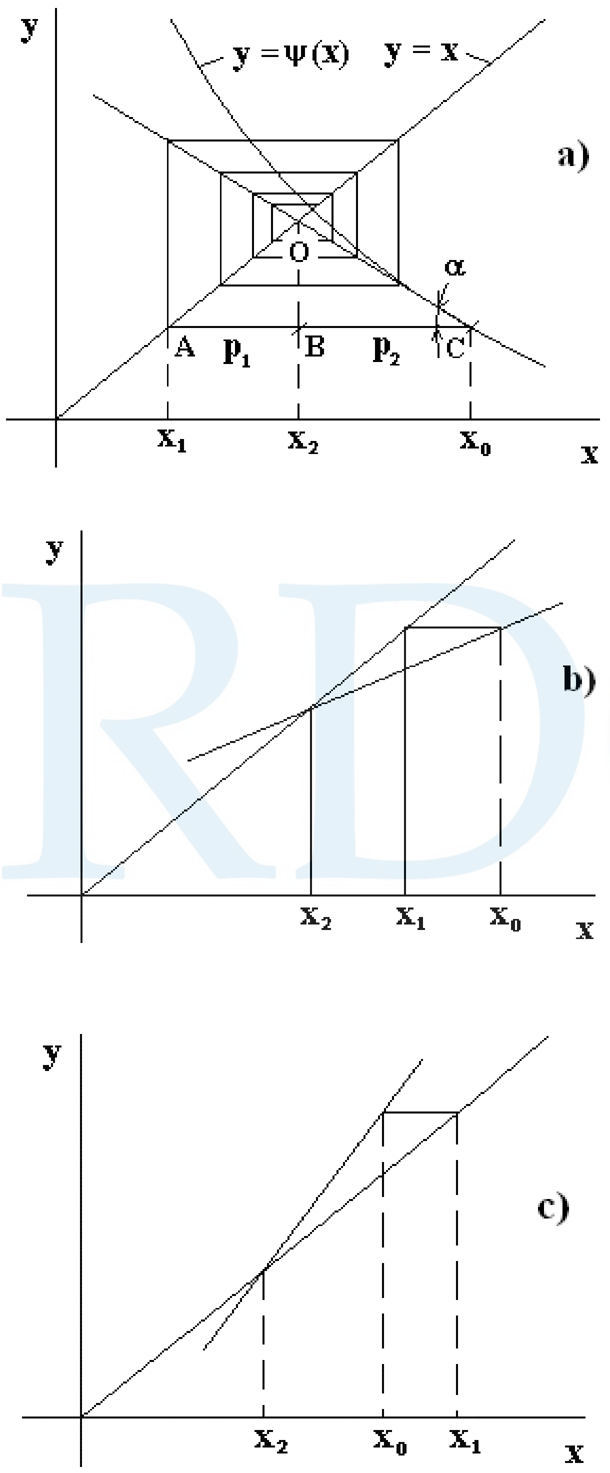

Fig. 1. Geometrical interpretation of convergence of the GMI

The presence of the variable parameter $\lambda \in \boldsymbol{R}$ in the formula (3.1), which is selected by one single criterion of fastest possible convergence of numerical solutions, as it will be seen from examples, extends the field of application of the presented iteration process. It has been called *generalize method of iteration*.

Let us consider the geometrical interpretation of the GMI, as a result of which, in particular, is specified the parameter λ.

For points sufficiently close to the root $x = \xi$, where the assimilation of the tangent at the points x_0, x_1 and x_2 is allowed, i.e. $\psi'(x_0) = \psi'(x_1) = \psi'(x_2) = \mathrm{tg}\alpha$, for the spiral iteration in fig. 1, (a), from the one side the fallowing equalities take place:

$$\frac{BO}{BC} = \frac{AB}{BC} = \frac{p_1}{p_2} = |\mathrm{tg}\alpha|, \quad \text{or} \quad x_2 - x_1 = (x_0 - x_2)\mathrm{tg}\alpha, \quad \mathrm{tg}\alpha < 0.$$

From the other side, for the steps (fig. 1, b) and an hyper-steps (fig. 1, c) iterations is valid the relation

$$x_1 - x_2 = (x_0 - x_2)\mathrm{tg}\alpha, \quad \mathrm{tg}\alpha > 0.$$

Consequently, taking into account the notation $\mathrm{tg}\alpha = 1 - \lambda$, the assumption on the completeness of the metric space $\boldsymbol{R}$ is confirmed, and with that the justification of the initial expression (3.1) for the generalize contraction mapping assuming the validity of the condition of existence and uniqueness of *fixed-point* (the theorem of Banach) in $\boldsymbol{R}$ [6], which allows to reformulate the operator of the nonlinear contraction mapping in the final form:

$$\Phi(x) = \frac{\psi(x) - \psi'(x)x}{1 - \psi'(x)}. \tag{3.8}$$

As a nonlinear method of iteration (NMI) here and later well be implied the particular case of the GMI, when the parameter λ is determined by the following formula:

$$\lambda = 1 - \psi'(x). \tag{3.9}$$

For the cases (a) and (b) we get:

$$\lambda > 0 \qquad \left|\frac{\lambda - 1 + \psi'(x)}{\lambda}\right| < 1 \Leftrightarrow \begin{cases} -1 < \dfrac{\lambda - 1 + \psi'(x)}{\lambda} \\ \dfrac{\lambda - 1 + \psi'(x)}{\lambda} < 1 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \psi'(x) > -2|\lambda| + 1 & \text{(for spiral iteration [3] at } -2|\lambda| + 1 < \psi'(x) < 0\text{);} \\ \psi'(x) < 1 & \text{(for steps iteration [3] at } 0 < \psi'(x) < 1\text{).} \end{cases}$$

For the case (c) we get:

$$\lambda < 0 \qquad \left|\frac{\lambda - 1 + \psi'(x)}{\lambda}\right| < 1 \Leftrightarrow \begin{cases} -1 < \dfrac{-\lambda - 1 + \psi'(x)}{-\lambda} \\ \dfrac{-\lambda - 1 + \psi'(x)}{-\lambda} < 1 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \psi'(x) < 2|\lambda| + 1; & \text{(for hyper-steps iteration [3]} \\ \psi'(x) > 1. & \text{at } 1 < \psi'(x) < 2|\lambda| + 1\text{);} \end{cases}$$

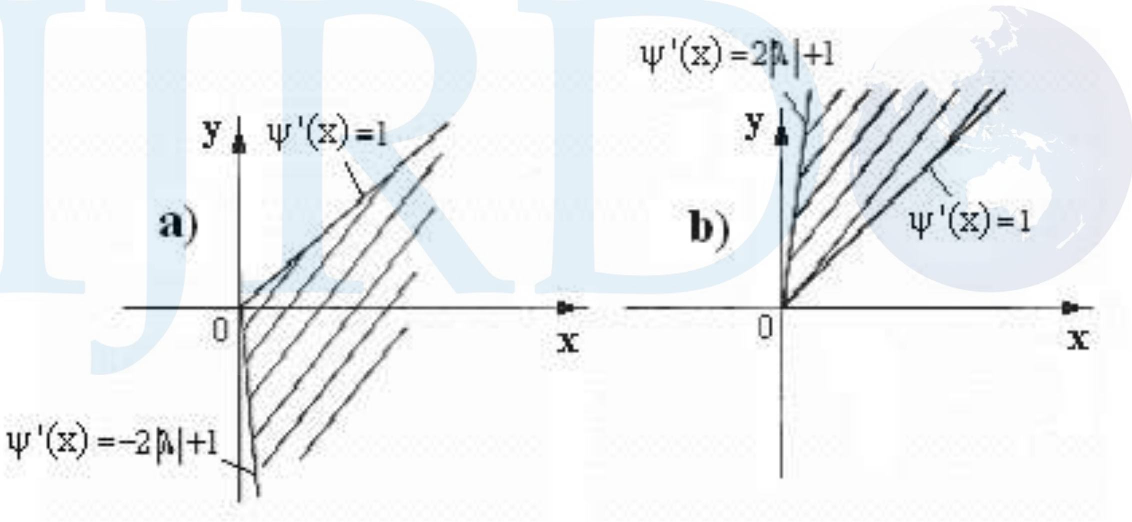

Fig. 2. Representation of convergence sub-regions of the GMI

So, from the above-mentioned derivations is becomes clear, that taking a sufficiently large by the absolute value number $|\lambda| \in R$, i.e. for $|\lambda| \to \infty$, all three sub-regions of the convergence [4] (fig. 2, a and b) covered practically all the spectrum of possible variations of the tangents on the plane, which was required to prove. For here it follows, that the operator $\Phi(x)$ is a *nonlinear-generalize contraction mapping* with *contractible original* $\psi(x)$ (fig. 3).

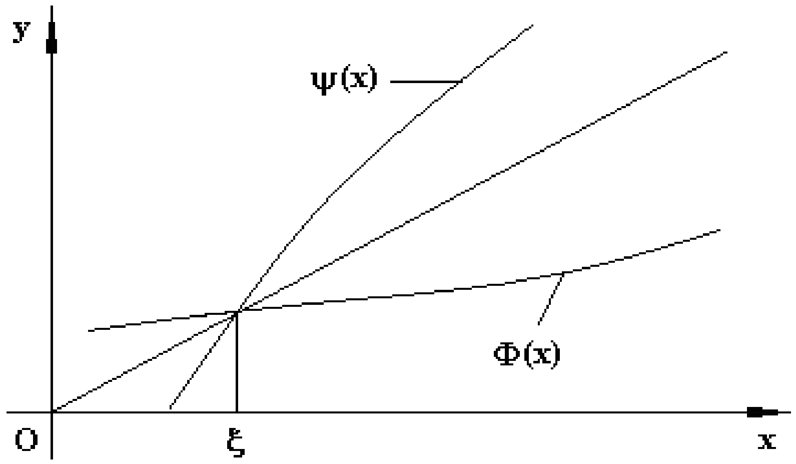

Fig. 3. Nonlinear-generalize contraction mapping $\Phi(x)$ **with** *contractible original $\psi(x)$*

4. Nonlinear-generalize contraction mapping as a function from some variables.

In the space $\boldsymbol{E_n}$ one introduce a canonic norm $\|\vec{x}\|$, expressed by one of the following norms [7]:

$$\|\vec{x}\|_k = \sqrt{\sum_{i=1}^{n} x_i^2}, \quad \text{or} \quad \|\vec{x}\|_l = \sum_{i=1}^{n} |x_i|, \quad \text{or} \quad \|\vec{x}\|_m = \max_i |x_i|.$$

In this case the mapping

$$\|\vec{\Phi}(\vec{x}_1) - \vec{\Phi}(\vec{x}_2)\| \le q\|\vec{x}_1 - \vec{x}_2\|, \quad 0 < q < 1$$

can be classified as a nonlinear-generalize one in the space $\boldsymbol{E_n}$. Therefore, by the introduced operator of nonlinear mapping $\vec{\Phi}$ in $\boldsymbol{E_n}$ all the known theorems will be formally valid out [7], proofed for the classical operator of the contractible mapping $\vec{\varphi}$ in $\boldsymbol{E_n}$.

On the base of the forgoing the formalization is allowed for the nonlinear operator of the contraction mapping $\vec{\Phi}$ in the space $\boldsymbol{E_n}$ as an operator, providing the contraction of an arbitrary contractible original $\vec{\psi}$ to the fixed-point, i.e.

$$\vec{\Phi}(\vec{x}) = [\vec{\Lambda}(\vec{x})]^{-1} \cdot \{\vec{\psi}(\vec{x}) + [\vec{\Lambda}(\vec{x}) - \vec{E}] \cdot \vec{x}\}, \quad \text{where}$$

$$\left[\begin{array}{l} \vec{\Lambda}(\vec{x}) = \pm \mathrm{Diag}_{i=1,\dots,n}\left(\lambda_i = |\lambda|\right) = \text{const} \quad \text{for the GMI;} \\ \quad \vec{\Lambda}(\vec{x}) = \vec{E} - \vec{\psi}'(\vec{x}) \ne \text{const} \quad \text{for the NMI,} \end{array}\right. \tag{4.1}$$

where $\vec{E}$ is the unit matrix and $\vec{\psi}'(\vec{x})$ is the Jacobs matrix of the vector-function for the contractible original $\vec{\psi}(\vec{x})$, namely,

$$\vec{\Psi}(\vec{x}) = \vec{\psi}'(\vec{x}) = \begin{bmatrix} \frac{\partial\psi_1}{\partial x_1} & \frac{\partial\psi_1}{\partial x_2} & \cdots & \frac{\partial\psi_1}{\partial x_n} \\ \frac{\partial\psi_2}{\partial x_1} & \frac{\partial\psi_2}{\partial x_2} & \cdots & \frac{\partial\psi_2}{\partial x_n} \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\partial\psi_n}{\partial x_1} & \frac{\partial\psi_n}{\partial x_2} & \cdots & \frac{\partial\psi_n}{\partial x_n} \end{bmatrix}. \tag{4.2}$$

It is known [8], that in the two forms of the presentation of the system of equations $\vec{\psi}(\vec{x}) = \vec{x}$ and $\vec{f}(\vec{x}) = 0$ the vector-functions $\vec{\psi}(\vec{x})$ and $\vec{f}(\vec{x})$ are related by the transformations:

$$\begin{cases} \vec{\psi}(x) = \vec{f}(x) + \vec{x}; & (4.3) \\ \vec{\psi}'(x) = \vec{f}'(x) + \vec{E}, & (4.4) \end{cases}$$

whereby the transition is realized from the formulation of the vector-operator of the contraction mapping $\vec{\Phi}$ valuated to the statement of the problem on the fixed-point to the formulation of the same nonlinear-generalize operator $\vec{\Phi}$ in the space $\boldsymbol{E_n}$ in the context of the solution of the problem about the zeros.

Consequently,

$$\vec{\Phi}(\vec{x}) = \vec{x} - [\vec{f}'(\vec{x})]^{-1} \cdot \vec{f}(\vec{x}) = \vec{x} - [\vec{J}(\vec{x})]^{-1} \cdot \vec{f}(\vec{x}), \tag{4.5}$$

where $\vec{J}(\vec{x})$ is the Jacobs matrix of the vector-function $\vec{f}(\vec{x})$, from which, by assuming the fundamental equality $\vec{\Phi}(\vec{x}) = \vec{x}$, it can be ascertained in the coincidence of the expressions for the vector operator of the nonlinear-generalize contraction mapping $\vec{\Phi}$ and for the principal recursive dependence of the classical method of Newton for the system of equations in real roots [7]:

$$\vec{x}_{k+1} = \vec{x}_k - [\vec{J}(\vec{x}_k)]^{-1} \cdot \vec{f}(\vec{x}_k) \cong \vec{x}_k - [\vec{J}(\vec{x}_0)]^{-1} \cdot \vec{f}(\vec{x}_k), \quad k = 1,2,\ldots,. \tag{4.6}$$

5. Nonlinear-generalize mapping on the complex plane. On the bases of the preliminary formula (3.1) it is not difficult to conclude, that the parameter λ in the expression for the scalar operator of the nonlinear-generalize mapping (3.8) one take not only real, but also complex values. So, from the mathematical point of view it is completely legitimate definition of the scalar operator for nonlinear-generalize contraction mapping Φ^* in the complex space $\boldsymbol{C}$:

$$\Phi^* = \Phi(x^*) = \frac{\psi(x^*) + (\lambda^* - 1)x^*}{\lambda^*}, \quad \text{where} \quad \lambda^* = 1 - \psi'(x^*) \tag{5.1}$$

with

$$x^* = x^r + ix^i, \quad \psi(x^*) = y^r + iy^i \quad \text{and} \quad \lambda^* = \lambda^r + i\lambda^i, \quad \text{where} \quad i = \sqrt{-1}.$$

Then the iteration process NIM for the computation of the complex root of the equation $\psi(x) = x$ will be expressed by the following recursive dependence:

$$x^*_{k+1} = \frac{\psi(x^*_k) + (\lambda^*_k - 1)x^*_k}{\lambda^*_k}, \quad k = 0,1,2,\dots, \tag{5.2}$$

which is equivalent to the system of two independent recursive equations, i.e.

$$\begin{cases} x^r_{k+1} = x^r_k + \dfrac{(y^r_k - x^r_k)\lambda^r_k + (y^i_k - x^i_k)\lambda^i_k}{(\lambda^r_k)^2 + (\lambda^i_k)^2}; \\ x^i_{k+1} = x^i_k + \dfrac{(y^i_k - x^i_k)\lambda^r_k - (y^r_k - x^r_k)\lambda^i_k}{(\lambda^r_k)^2 + (\lambda^i_k)^2}, \end{cases} \quad k = 0,1,2,\dots, \tag{5.3}$$

or, in the matrix form of the presentation,

$$\vec{x}^*_{k+1} = \vec{x}^*_k + [\vec{\Lambda}(\vec{x}^*_k)]^{-1} \cdot [\vec{\psi}(\vec{x}^*_k) - \vec{x}^*_k], \quad k = 0,1,2,\dots, \quad \text{where}$$

$$\vec{x}^*_k = [x^r_k, x^i_k]^T, \quad \vec{\psi}(\vec{x}^*_k) = [y^r_k, y^i_k]^T \quad \text{and} \quad \vec{\Lambda}^{-1}_k = \frac{1}{(\lambda^r_k)^2 + (\lambda^i_k)^2}\begin{bmatrix} \lambda^r_k & \lambda^i_k \\ -\lambda^i_k & \lambda^r_k \end{bmatrix}. \tag{5.4}$$

As in the case for real roots, the formulation of NIM for the case of complex roots by taking into account the transformations

$$\begin{cases} \psi(x^*) = f(x^*) + x^*; & (5.5) \\ \psi'(x^*) = f'(x^*) + 1 & (5.6) \end{cases}$$

is also reduced to the main recursive dependence of the Newton method for the complex roots [7]:

$$\vec{x}^*_{k+1} = \vec{x}^*_k - \vec{J}^{-1}(\vec{x}^*_k)\cdot\vec{f}(\vec{x}^*_k), \quad k = 0,1,2,\ldots. \tag{5.7}$$

6. Aggregate method of iteration. Now we can formulate an *aggregate method of iteration* (AIM) composed on the base of the formulation of the GMI and the NMI for the problem about the zeros of the numerical solution of nonlinear algebraic and transcendental equations systems [3]:

$$\vec{x}_{k+1} = \frac{\vec{\psi}(\vec{x}_k) + (\lambda - 1)\vec{x}_k}{\lambda}, \quad k = 0,1,2,\ldots \quad \text{for the GMI;} \tag{6.1}$$

$$\vec{x}_{k+1} = [\vec{\Lambda}(\vec{x}_k)]^{-1}\cdot\{\vec{\psi}(\vec{x}_k) + [\vec{\Lambda}(\vec{x}_k) - \vec{E}]\cdot\vec{x}_k\}, \text{ where } \vec{\Lambda}(\vec{x}_k) = \vec{E} - \vec{\psi}'(\vec{x}_k) \quad k = 0,1,2,\ldots \text{ for the NMI.} \tag{6.2}$$

7. Examples of the numerical realization. As a first example we consider the case of the numerical solution of the transcendental equation in complex roots, where the problem of the stability of a multi layered base is investigated [9]:

$$\alpha p\,\mathrm{tg}p = 1 + i\beta p, \text{ where } p = p_1 + ip_2, \tag{7.1}$$

where α and β are parameters, taking the following values: $\alpha = 0,\,1,\,10$ and $\beta = 0;\,0{,}1$. The equation (7.1) in complex roots can be transformed to the system of two transcendental equations for real roots:

$$\begin{cases} \alpha p_1\,\mathrm{tg}p_1 - \alpha p_2\,\mathrm{th}p_2 = 1 + \beta p_1\,\mathrm{tg}p_1\,\mathrm{th}p_2 - \beta p_2, \\ \alpha p_1\,\mathrm{th}p_2 + \alpha p_2\,\mathrm{tg}p_1 = \beta p_1 - \mathrm{tg}p_1\,\mathrm{th}p_2 + \beta p_2\,\mathrm{tg}p_1\,\mathrm{th}p_2. \end{cases} \tag{7.2}$$

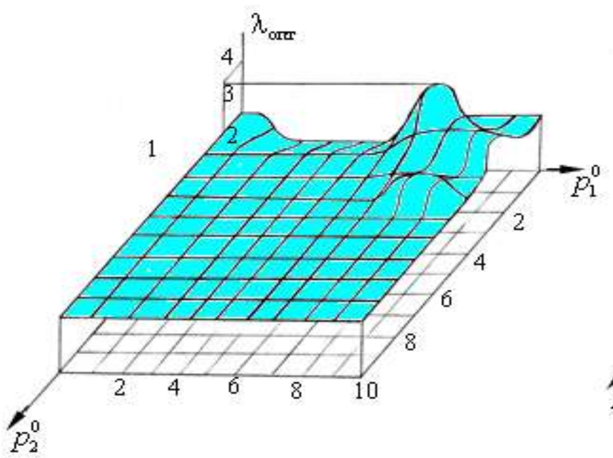

Fig. 4.

Dependence λ_{opt} ***on initial approximation with*** $\alpha = 0{,}1;\ \beta = 0$

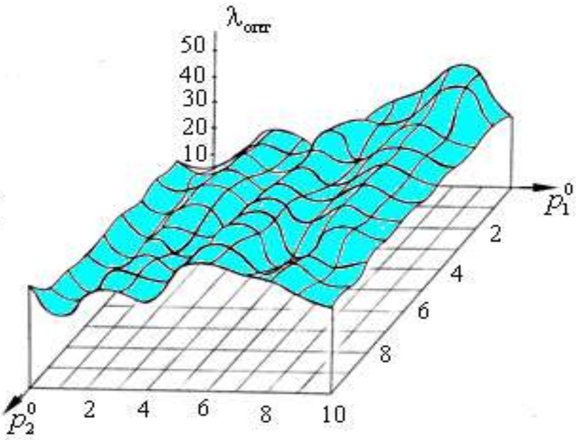

Fig. 5.

Dependence λ_{opt} ***on initial approximation with*** $\alpha = 10;\ \beta = 0{,}1$

In fig. 4 and 5 the optimal values of the parameter λ_{opt} are given for the achieving of convergence of numerical results of the presented example solved by the GIM for complex roots in dependence of initial approximation p_1^0 and p_2^0. It is necessary to note that in the calculation the simplification $\lambda^* = \lambda^r + i\lambda^i \approx \lambda^r$ is accepted which does not lead out the error of numerical calculations beyond 5% the accuracy of the generation of numerical results is of numerical results is $\varepsilon = 10^{-5}$. Similar tables one can obtained for arbitrary pairs of the values of α and β. It's also necessary to note that the calculations of the examples by the NIM (Newton method) for the complex root generates the results, not exceeding by convergence velocity the results, indicated in fig. 4 and 5, its error is near 5%.

It is necessary to pay the attention on the circumstance, that the attempt to solve the test example by the Newton method (GMI) for real roots and by other methods of numerical solution do not lead to satisfactory results to find the roots.

As a second example of the algorithmic solution of a system of nonlinear equations we choose the system of quasi transcendental equations, i.e. of the equation, having an analytical solution, for some particular case of the Collatz conjecture (conjecture of Syracuse) [10].

This system of equations, assuming for $k_0 = 13, k_1 = 5, k_2 = 1$ the single solution in natural numbers, has the following form:

$$\begin{cases} 3k_0 - 2^{\eta_1} k_1 + 1 = 0; \\ 3k_0 + 3k_1 - 2^{\eta_1} k_1 - 2^{\eta_2} k_2 + 2 = 0. \end{cases} \tag{7.3}$$

In fig. 6 and 7 the dependences the number of iterations is given on initial approximation of the parameters η_1 and η_2 for the indicated system of equations for $\lambda = 20$ and $\varepsilon = 10^{-10}$. Along with this in fig. 8 the surface is illustrated characterizing the ratio of the iteration numbers, generated on the base of NIM (Newton method) and AIM respectively.

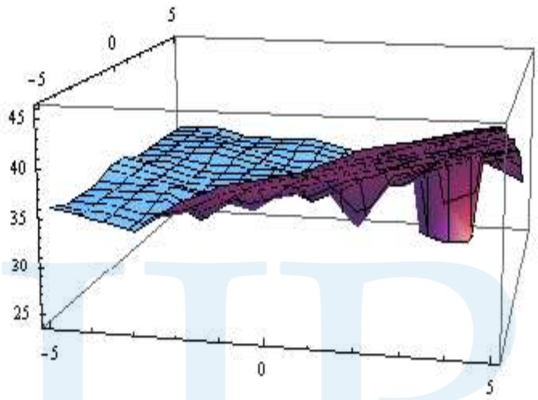

Fig. 6.
Dependence iteration numbers
on original approximation by AMI

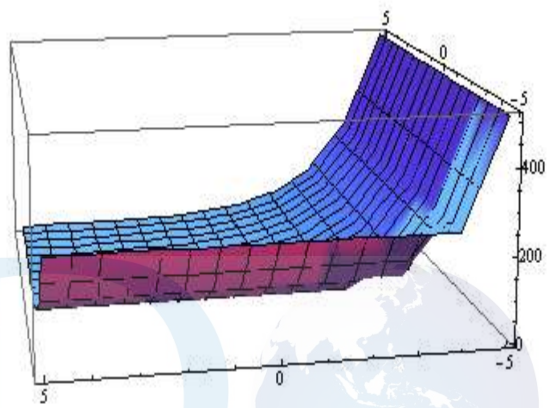

Fig. 7.
Dependence iteration numbers
on original approximation by NIM

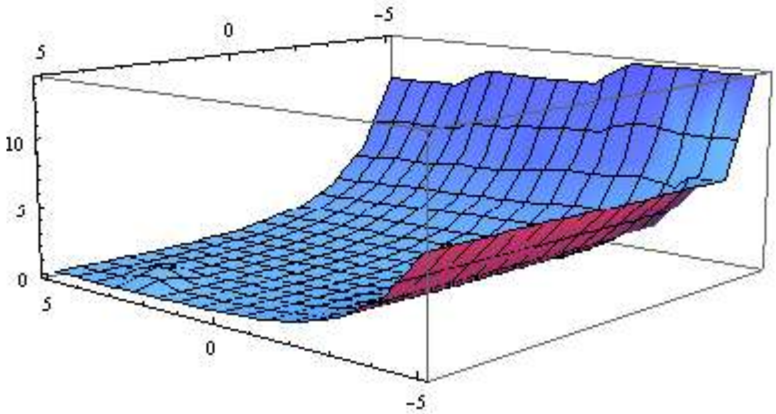

Fig. 8. Ratio of the iteration numbers by NIM and AMI

The offered problem solved for the three quasi transcendental equations also, for which is bring to light to the some order of the advance by convergence velocity of NIM and AIM, which is illustrated on the fig. 8 for the case of two equations.

8. Conclusion. The generalize method of iteration presented as a hybrid of a classic method of iteration and of the method of proportional division of the interval, on which satisfied the conditions of the theorem of Bolzano-Cauchy, is formulated. The proof of the convergence of proposed algorithm is brought. At first, the operator of the contraction mapping as a function from one variable is considered. The theorem about convergence giving iteration and its evidence are shown. The cases as the steps, spiral and also an hyper-steps iterations were considered. A geometrical illustration of the GMI was finally exposed.

The nonlinear iteration method is formulated on the base of the obtained nonlinear operators of the scalar and vector contraction mapping as a function from one and also for any finite number of several real variables. Also the formulation of the nonlinear-generalize contraction mapping in the complex space is taking into account. It is shown that the NIM is equivalent to the classical Newton method for real and complex roots.

Finally, the aggregate method of iteration, composed by GMI and NMI, is formulated for the problem about the zeros as the most effective nonlinear algebraic and transcendental equations systems numerical solutions hybrid strategy.

On the basis of solution of the two nonlinear equations systems of the transcendental equations the expected efficiency of the AIM with respect to other known methods is proved.

References

1. **Vantsyan A.A**. About one method of solution of the transcendent and algebraic equations // Information Technologies and Management. – 2004. – N4. – P. 44 – 47.
2. **Vantsyan A.A., Gevorgyan H.A.** Modernize method of iteration // The mechanics of a composite materials and constructions, Proc. of IV – All Russian symposium. – Moscow, 2012. – V. 1. – P. 40 – 44.
3. **Vantsyan A.A., Gevorgyan H.A.** About one method solution of transcendental and algebraic equations. – Yerevan: Gitutyun, 2014. – 56 p.
4. **Vantsyan A.A., Gevorgyan H.A., Moravej M.** Comparison analysis of a generalize method of iteration on the basis of solutions of some problems of the mechanics // Research Journal of Fisheries and Hydrobiology, – 10(10) June 2015, P. 772 – 775.
5. **Vantsyan A.A.** Generalize method of iteration // Vestnik RAEN, 2015, N4, P. 13 – 15.
6. **Kolmogorov A.N., Fomin S.V.** The elements of the theory of functions and functional analysis. – Moscow: Nauka, 1976. – 542 p.
7. **Demidovich B.P., Maron I.A.** Basis of the computational mathematics. – Moscow: Nauka, 1970. – 664 p.
8. **Bakhvalov et al.** Numerical metods. – Moscow: Nauka, 1987. – 598 p.
9. **Movsisyan L.A.** About one method of the seismoprotection // Mechanics, Bulletin of the NAS RA. – 2011. – N4, V. 111. – P. 325 – 335.
10. **Hayes B.** Table off and fall of the numbers-hailstone. – Scientific American, 1984. – N3. – C. 102 – 107.

INFORMATION ABOUT AUTHORS

Vantsyan Anushavan A.,

Chief sci. researcher at the Institute of mechanics

of National Academy of Sciences of the Republic of Armenia,

doc. ph.-math. sci., professor, academician of RANS

Yerevan – 19, ave Marshal Baghramyan 24 B, Institute of Mechanics

tel. (374 10) 561 523 (off.), (374 93) 524 501 (mob.),

E – mail: vantsyan@mechins.sci.am

Gevorgyan Hrant A.,

Doctor of engineering, researcher at the Institute of mechanics

of National Academy of Sciences of the Republic of Armenia,

Yerevan – 19, ave Marshal Baghramyan 24 B, Institute of Mechanics

tel. (374 10) 561 523 (off.), (374 96) 390 315 (mob.),

E – mail: hrgevorkian@mail.ru