FUZZY COLORING OF FUZZY CYCLES USING MATLAB

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Abstract

This work investigates the chromatic number of fuzzy cycles and its middle graphs through a fuzzy coloring approach implemented in MATLAB. The methodology ensures proper fuzzy coloring and the outcomes are validated using computational results with graphical visualization.

Keywords: *Fuzzy Coloring, Chromatic Number, Fuzzy Cycle, Middle Graph, MATLAB.*

1. Introduction

Fuzzy graph coloring has become an important area of research in combinatorial optimization [1], as it provides effective methods for addressing problems characterized by vagueness, imprecision, and uncertainty. It plays a crucial role in resource allocation and task scheduling, ensuring conflict-free schedules and optimal resource utilization [2]. To address such situations, Susana Munoz et al. [3] introduced fuzzy graph coloring in 2005 by reformulating the traffic light problem in terms of fuzzy graphs, focusing on crisp vertex sets with fuzzy edge sets (type–1 fuzzy graphs). In 2006, Eslahchi and Onagh [4] extended this framework to type–2 fuzzy graphs, where both vertices and edges are fuzzy, and proposed a coloring method based on strong adjacency relations. A further development came in 2015 when Sovan Samanta et al. [5] introduced fuzzy colors, in which the coloring of vertices depended on the strength of their incident edges, and they also defined the fuzzy chromatic number. Building upon these contributions, the present work aims to extend the fuzzy coloring procedure by integrating fuzzy colors and edge strengths into a more generalized framework. In 2024, we determined the chromatic number of certain families of fuzzy graphs using fuzzy colors based on the strength of an edge incident to a vertex, and derived various properties of fuzzy coloring [6]. This work was later extended to compute the chromatic number of some related graphs of fuzzy paths, fuzzy cycles and homogeneous fuzzy caterpillar (refer [7,8,9]).

MATLAB (MATrix LABoratory) was originally developed as a user-friendly tool for linear algebra and eigenvalue problems, but it has since evolved into a comprehensive platform for scientific computing and research [10]. As a high-performance computing language, MATLAB combines numerical analysis, visualization, and a modern programming environment into a unified system [11]. This integration allows researchers and educators to move seamlessly from theoretical models to computational experiments and graphical interpretation of results. One of its key strengths lies in its interactive nature: arrays form the core data type, which simplifies coding and accelerates problem-solving compared to conventional programming languages. In addition to its computational power, MATLAB incorporates debugging tools, object-oriented programming support, and an extensive library of specialized toolboxes that address applications in engineering, mathematics, optimization, and simulation [12]. These features have established MATLAB not only as a computational tool but also as an effective platform for teaching, learning, and innovation across multiple disciplines. In this study, we are trying to develop a MATLAB coding to determine the chromatic number of fuzzy cycles and its middle graph using the fuzzy coloring procedure [6].

The structure of this paper is as follows. Section 1 introduces fuzzy coloring and provides an overview of MATLAB. Section 2 presents the key definitions and preliminary concepts necessary for the study. In Section 3, we develop MATLAB code to determine the chromatic number of fuzzy cycles by using the fuzzy coloring procedure.. Section 4, focuses on the implementation of MATLAB code to compute the chromatic number of middle graph of a fuzzy cycle by using the fuzzy coloring procedure.

2. Preliminaries

This section recalls the theoretical foundations of fuzzy graph theory and fuzzy coloring, which are essential for analyzing and computing the chromatic number of fuzzy cycles and its middle graph within MATLAB.

Definition 2.1. [13] A fuzzy graph $G = (V, \sigma, \mu)$ is a pair of functions (σ, μ), where $\sigma : V \to [0, 1]$ is a fuzzy subset of a non-empty set V, and $\mu : V \to [0, 1]$ is a symmetric fuzzy relation on σ, such that the relation $\mu(v_i, v_j) \le \sigma(v_i) \wedge \sigma(v_j)$ is satisfied for all $v_i, v_j \in V$ and $(v_i, v_j) \in E \subset V \times V$.

Here, $\sigma(v_i)$ denote the degree of membership of the vertex v_i, and $\mu(v_i, v_j)$ denotes the degree of membership of the edge relation $e_{ij} = (v_i, v_j)$ on $V \times V$.
Note : In this paper, we denote $\sigma(v_i) \wedge \sigma(v_j) = min\{\sigma(v_i), \sigma(v_j)\}$, and $(v_i) \vee \sigma(v_j) = max\{\sigma(v_i), \sigma(v_j)\}$.

Definition 2.2. [14] Let $G = (V, \sigma, \mu)$ be a fuzzy graph with underlying crisp graph G^*. A fuzzy path P_n in G is a sequence of distinct vertices $v_0, v_1, \ldots, v_n$ such that $\mu(v_{i-1}, v_i) > 0, 1 \leq i \leq n$. Here $n \geq 1$ is called the length of the path P_n.

Definition 2.3. [14] A fuzzy path P_n in which $v_0 = v_n$ and $n \geq 3$, then P_n is called a fuzzy cycle C_n of length n.

Definition 2.4. [3] Let $G = (V, \sigma, \mu)$ be a fuzzy graph and an edge $e = (v_i, v_j) \in G$ is called **strong** if $\frac{1}{2}\{\sigma(v_i) \wedge \sigma(v_j)\} \leq \mu(v_i, v_j)$ and it is called **weak** otherwise.

Definition 2.5. [3] Let $G = (V, \sigma, \mu)$ be a fuzzy graph and the **strength of an edge** $(v_i, v_j) \in G$ is denoted by,

$$I(v_i, v_j) = \frac{\mu(v_i, v_j)}{\sigma(v_i) \wedge \sigma(v_j)}.$$

Definition 2.6. [15] A fuzzy graph $G = (V, \sigma, \mu)$ is called a strong fuzzy graph if each edge in G is a strong edge.

Definition 2.7. [6] Let $G = (V, \sigma, \mu)$ be a fuzzy graph. Fuzzy coloring is an assignment of basic or fuzzy colors to the vertices of a fuzzy graph G and it is proper,

(i) if two vertices are connected by a strong edge, then they either have different basic or fuzzy colors(if necessary), or one vertex can have a basic color and the other can have a fuzzy color corresponding to different basic color.

(ii) if two vertices are connected by a weak edge, then they either have same or different fuzzy colors, or one vertex can have a basic color and other can have a fuzzy color corresponding to the same basic color.

Definition 2.8. [6] Let $G = (V, \sigma, \mu)$ be a fuzzy graph. Perfect fuzzy coloring (optimal fuzzy coloring) is an assignment of minimum number of colors (basic or fuzzy) for a proper fuzzy coloring of G.

Definition 2.9. [6] Let $G = (V, \sigma, \mu)$ be a fuzzy graph. The minimum number of colors (basic or fuzzy) needed for a proper fuzzy coloring of G is called the chromatic number of G and is denoted by $\chi_f(G)$.

Definition 2.10. [7] The middle graph $M_f(G)(V_M, \sigma_M, \mu_M)$ of a fuzzy graph $G(V, \sigma, \mu)$ is a fuzzy graph with underlying crisp graph $M(G)(V_M, E_M)$, with the vertex set $V_M = V \cup V_{ij}$ where $V = \{v_i \mid v_i \in V\}$ and $V_{ij} = \{v_{ij} \; \forall \; (v_i, v_j) \in E\}$ and $\nu(M_f(G)) = n + 1 + n = 2n + 1$ and the edge set

$$E_M = \begin{cases} (v_{ij}, v_i), (v_{ij}, v_j) & \forall \; i \; and \; j, \\ (v_{ij}, v_{rs}) & if \; the \; edges \; (v_i, v_j) \; and \; (v_r, v_s) \; are \; adjacent \; in \; G. \end{cases}$$

Then, $\sigma_M(v_i) = \sigma(v_i) \; if \; v_i \in V, 0 \leq \; i \leq \; n,$
$\sigma_M(v_{ij}) = \mu(v_i, v_j) \; if \; (v_i, v_j) \in E \; \forall \; \; i \; and \; j,$
$\mu_M(v_{ij}, v_{rs}) = \mu(v_i, v_j) \wedge \mu(v_r, v_s)$ if the edges (v_i, v_j) and (v_r, v_s) are adjacent in G,
and $\mu_M(v_i, v_{ij}) = \mu_M(v_j, v_{ij}) = \mu(v_i, v_j) \; \forall \; i \; and \; \; j.$

Lemma 2.1. [6] *Let C_n be a fuzzy cycle of length n. If all edges are weak in C_n, then* $\chi_f(C_n) = 1$.

Lemma 2.2. [6] *Let C_n be a fuzzy cycle of length n. If all the edges are strong in C_n, then*

$$\chi_f(C_n) = \begin{cases} 2 \; \; if \; n \; is \; even, \\ 3 \; \; if \; n \; is \; odd. \end{cases}$$

Theorem 2.1.[8] *If $M_f(C_n)$ is a strong fuzzy graph, then $\chi_f(M_f(C_n)) = 3$.*

Theorem 2.2. [7] $\chi_f(G) \geq \; max\,\{\chi_f(G_i) :\; 1 \leq \; i \leq \; k\}$, *where* $G = G_1 \cup G_2 \cup \ldots \cup G_k$ *and* $G_i, 1 \leq i \leq k$ *are fuzzy graphs.*
Corollary 2.2.1. [7] $\chi_f(G) \geq max\,\{\chi_f(G_i) : 1 \leq i \leq k\}$, *where* $G = G_1 \oplus G_2 \oplus \ldots \oplus G_k$ *and* $G_i, 1 \leq i \leq k$ *are edge disjoint fuzzy graphs.*

3. Fuzzy Coloring of Fuzzy Cycle Using MATLAB

This study implements fuzzy coloring of fuzzy cycles using MATLAB. Here, we are trying to develop code to determine the chromatic number of a fuzzy cycle when all edges are either strong or weak. The procedure is divided into five main steps: Input of the number of vertices and edges with membership values, Edge classification, Fuzzy coloring, Computation of the chromatic number of the fuzzy cycle, and Visualization.

Step 1: Input of Membership Values

Let $\boldsymbol{C_n}$ be a fuzzy path graph with $\boldsymbol{n}$ vertices and $\boldsymbol{n}$ edges. The membership values of the vertices are denoted by $\boldsymbol{\sigma(v_i)}$, where $\boldsymbol{1 \leq i \leq n}$, and the membership values of the edges are denoted by $\boldsymbol{\mu(v_i, v_{i+1})}$, where $\boldsymbol{1 \leq i \leq n}$ with $\boldsymbol{v_{n+1} = v_1}$. These values are taken as input from the user.

1. Each vertex v_i is assigned a vertex membership value $\sigma(v_i)$.
2. Each edge (v_i, v_{i+1}) is assigned an edge membership value $\mu(v_i, v_{i+1})$.

Step 2: Edge Classification

Each edge is classified as strong or weak using the following rule: If

$$\frac{1}{2}\{\sigma(v_i) \wedge \sigma(v_j)\} \leq \mu(v_i, v_j),$$

then (v_i, v_{i+1}) is **strong**; otherwise, it is **weak**. The classification results are displayed for all edges in the fuzzy path.

Step 3: Fuzzy Coloring of Vertices

Coloring proceeds sequentially from vertex v_1 to v_n under the following rules:

1. **Case 1 – All edges are weak:**
   - The first vertex v_1 is assigned a basic color with full intensity.
   - Remaining vertices receive the same color with diluted intensity:

$$Intensity = 1 \; - \; \frac{\mu\big(v_i, v_j\big)}{\sigma(v_i) \wedge \sigma\big(v_j\big)}.$$

2. **Case 2 – All edges are strong:**
   - **Subcase 2.1 – $\boldsymbol{n}$ is even:** Assign two basic colors alternately around the cycle.
   - **Subcase 2.2 – $\boldsymbol{n}$ is odd:** Assign two basic colors alternately to vertices v_1 to v_{n-1}, and assign a third color to v_n.

Step 4: Computation of the Chromatic Number

The chromatic number $\chi_f(C_n)$ is computed. This value reflects the minimum number of colors (basic or fuzzy) required for proper fuzzy coloring.

Step 5: Visualization

The fuzzy path C_n is visualized in MATLAB:

1. Vertices are plotted as circles, colored according to their assigned fuzzy color and diluted based on intensity.
2. Edges are drawn as black lines, and their membership value μ is displayed at the midpoint.
3. Each vertex is labeled in the form : $(v_i, \sigma(v_i))/(color, indensity)$.

This visualization provides a clear and intuitive representation of fuzzy coloring, showing both the numerical and graphical interpretation of the fuzzy cycle.

Example 1. Figure 1 presents the MATLAB output for the fuzzy coloring of the fuzzy cycle C_3, in which all edges are weak. The vertices v_1, v_2, v_3 have membership values $0.9, 0.8, 0.7$, while the edges $(v_1, v_2), (v_2, v_3), (v_3, v_1)$ have membership values $0.39, 0.34, 0.34$.

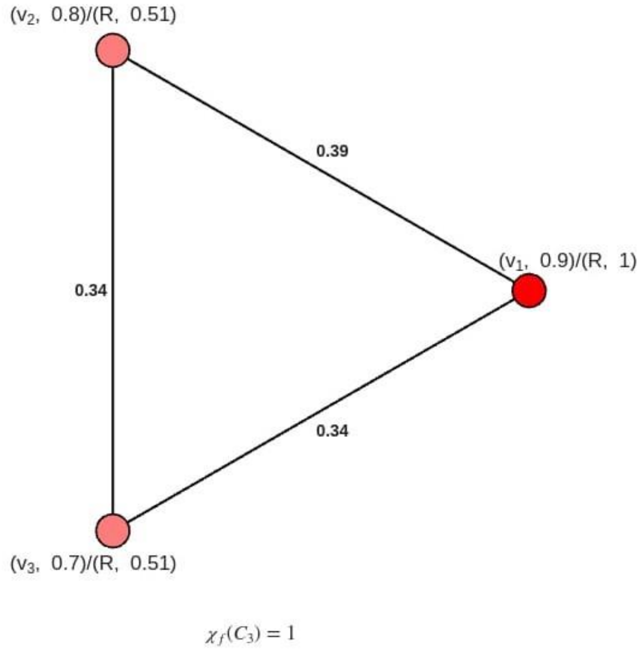

Figure 1. Fuzzy Coloring of Fuzzy Cycle C_3, when all the edges are weak.

Example 2. Figure 2 presents the MATLAB output for the fuzzy coloring of the fuzzy cycle C_4, in which all edges are weak. The vertices v_1, v_2, v_3, v_4 have membership values $0.9, 0.8, 0.7, 0.6$, while the edges $(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)$ have membership values $0.39, 0.34, 0.23, 0.24$.

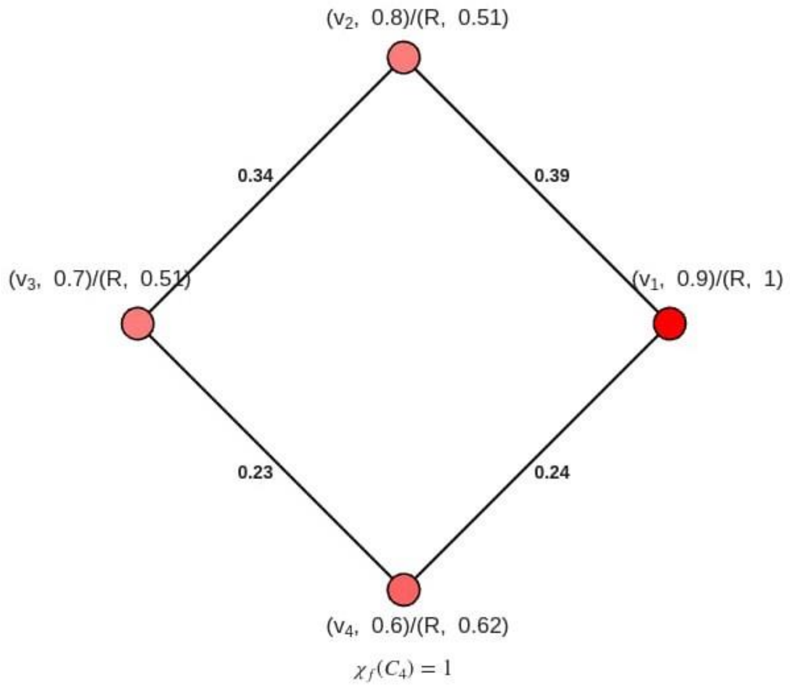

Figure 2. Fuzzy Coloring of Fuzzy Cycle C_4, when all the edges are weak.

Example 3. Figure 3 presents the MATLAB output for the fuzzy coloring of the fuzzy cycle C_4, in which all edges are strong. The vertices v_1, v_2, v_3, v_4 have membership values $0.9, 0.8, 0.7$, while the edges $(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)$ have membership values $0.6, 0.5, 0.6, 0.5$.

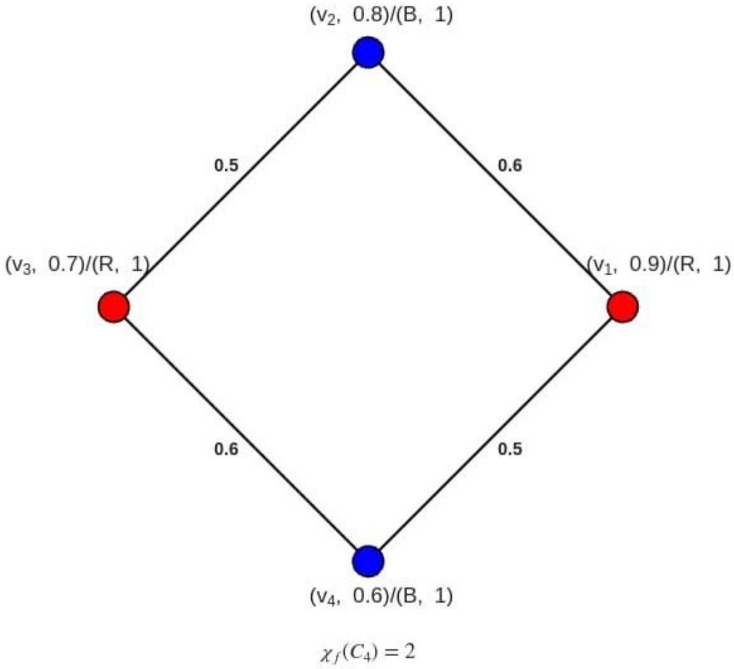

Figure 3. Fuzzy Coloring of Fuzzy Cycle C_4, when all the edges are strong.

Example 4. Figure 4 presents the MATLAB output for the fuzzy coloring of the fuzzy cycle C_3, in which all edges are strong. The vertices v_1, v_2, v_3 have membership values $0.9, 0.8, 0.7$, while the edges $(v_1, v_2), (v_2, v_3), (v_3, v_1)$ have membership values $0.6, 0.6, 0.6$.

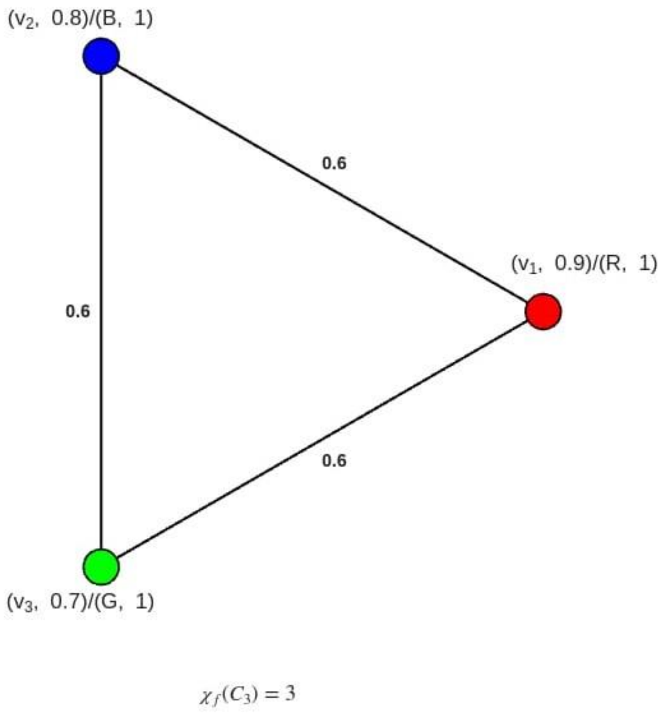

Figure 4. Fuzzy Coloring of Fuzzy Cycle C_3, when all the edges are strong.

4. Fuzzy Coloring of Middle Graph of Fuzzy Cycle Using MATLAB

This study implements the fuzzy coloring of the middle graph of a fuzzy cycle C_n using MATLAB. The procedure is divided into six main steps: Input of vertex and edge membership values, Construction of the middle graph of C_n, Edge classification, Fuzzy coloring of the vertices, Computation of the chromatic number, and Visualization of the resulting graph.

Step 1: Input of Membership Values

Let C_n be a fuzzy cycle with n vertices and n edges. The membership values of the vertices are denoted by $\sigma(v_i)$, for $i = 1,2, \dots, n$, and the membership values of the edges are denoted by $\mu(v_i, v_{i+1})$, for $i = 1,2, \dots, n$ (with $v_{n+1} = v_1$). These values are provided as input by the user.

Step 2: Construction of the Vertex Set

The middle graph $M_f(C_n)$ is constructed by introducing two categories of vertices:

1. All vertices of the fuzzy cycle C_n, namely $v_1, v_2, \dots, v_n$, each carrying its corresponding membership value $\sigma(v_i)$.
2. For each edge $(v_i, v_{i+1}) \in C_n$, a new vertex $v_i\ v_{i+1}$ is added. The membership value of this vertex is defined as $\sigma(v_{ii+1}) = \mu(v_i, v_{i+1})$.

Thus, the vertex set is given by, $V\left(M_f(C_n)\right) = \{v_1, v_2, \dots, v_n\} \cup \{v_1v_2, v_2v_3, \dots, v_{n-1}v_n\}$.

Step 3: Construction of the Edge Set

Two types of edges are introduced in $M_f(C_n)$:

1. For each edge (v_i, v_{i+1}) in C_n, the vertex v_{ii+1} is connected to both v_i and v_{i+1} with membership value $\mu(v_i, v_{ii+1}) = \mu(v_{i+1}, v_{ii+1}) = \mu(v_i, v_{i+1})$.
2. If two consecutive edges (v_i, v_{i+1}) and (v_{i+1}, v_{i+2}) exist in C_n, then the vertices v_{ii+1} and v_{i+1i+2} are connected with membership value
$$\mu(v_{ii+1}, v_{i+1i+2}) = min\{\mu(v_i, v_{i+1}), \mu(v_{i+1}, v_{i+2})\}.$$

Step 4: Edge Classification

Each edge (v_i, v_j) of $M_f(C_n)$ is classified as either strong or weak based on the following criterion:

$$\frac{1}{2}\{\sigma(v_i) \wedge \sigma(v_j)\} \leq \mu(v_i, v_j),$$

If the condition is satisfied, the edge is **strong**. Otherwise, the edge is **weak**.

Step 5: Coloring of the Middle Graph and the Chromatic Number

Since the middle graph $M_f(C_n)$ is a strong fuzzy graph, the adjacent vertex must be assigned a different basic color with full intensity. Also, the chromatic number $\chi_f(M_f(C_n))$ is computed. This value reflects the minimum number of colors (basic or fuzzy) required for proper fuzzy coloring.

Step 6: Visualization

Finally, the graph is visualized such that

1. Vertices are represented by circular nodes labeled with their names, membership values, and color-intensity pairs.
2. Edges are drawn between vertices with their membership values μ displayed at the midpoint.

This representation clearly highlights the structure of $M_f(C_n)$.

Example 5. Figure 5 presents the MATLAB output for the fuzzy coloring of middle graph of fuzzy cycle C_3. The vertices v_1, v_2, v_3 have membership values $0.9, 0.8, 0.7$, while the edges $(v_1, v_2), (v_2, v_3), (v_3, v_1)$ have membership values $0.4, 0.2, 0.4$.

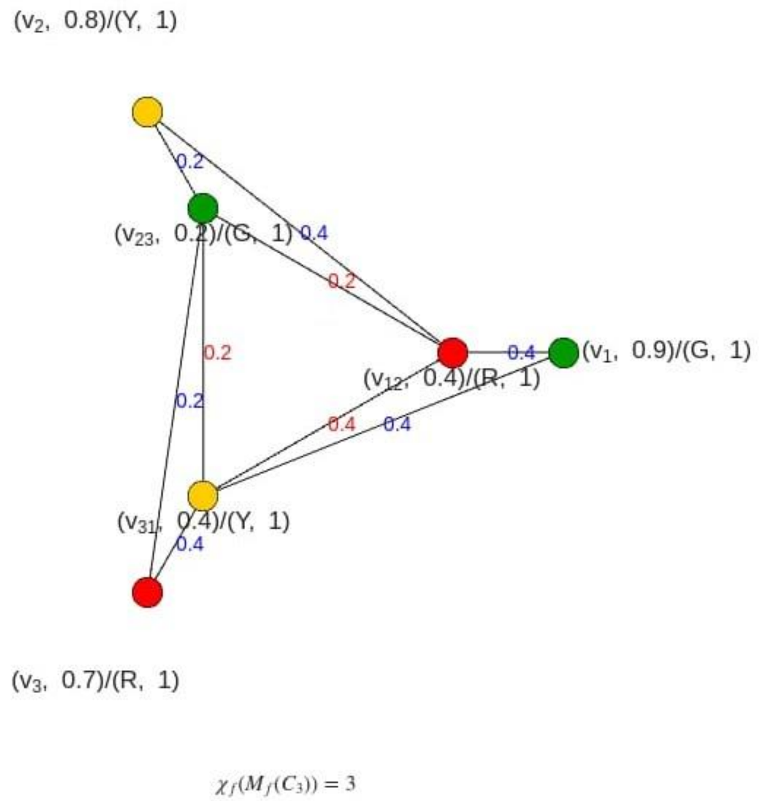

Figure 5. Fuzzy Coloring of Middle Graph $M_f(C_3)$.

Example 6. Figure 6 presents the MATLAB output for the fuzzy coloring of middle graph of fuzzy cycle C_4. The vertices v_1, v_2, v_3, v_4 have membership values $0.9, 0.8, 0.7, 0.6$, while the edges $(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)$ have membership values $0.4, 0.2, 0.4, 0.2$.

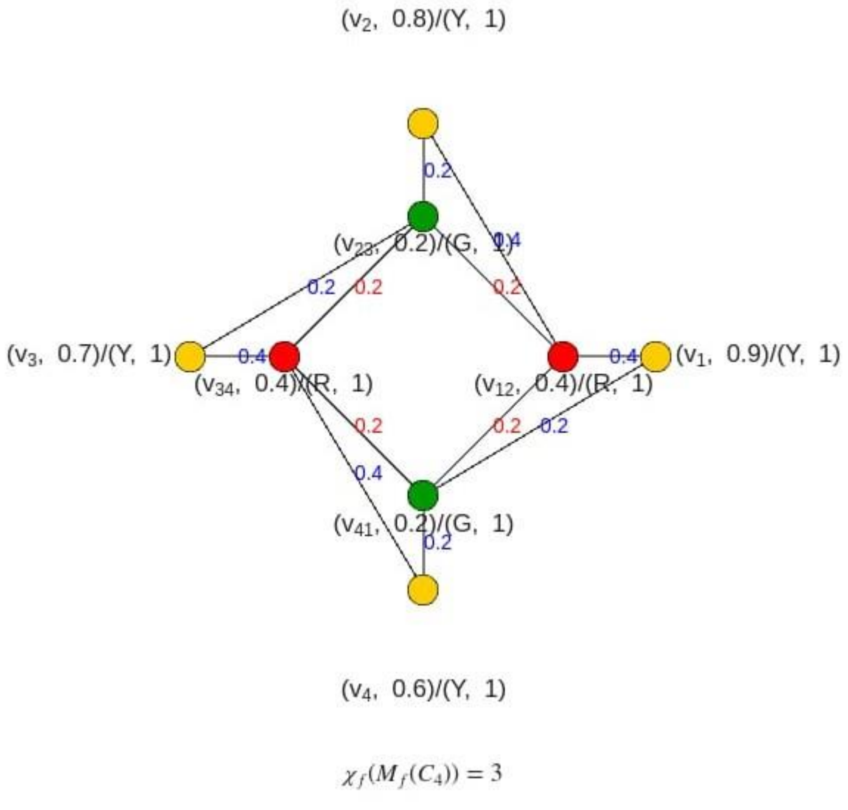

Figure 6. Fuzzy Coloring of Middle Graph $M_f(C_4)$.

5. References

[1] Kannan, M., Sathiragavan, M., Nivetha, P., Sankar, K., & Gurjar, J. (2024).Graph coloring techniques in scheduling and resource allocation. Journal of Nonlinear Analysis and Optimization, 15(2).

[2] Anjaly, K., & Sunitha, M. S. (2016). Chromatic number of resultant of fuzzy graphs. Fuzzy Information and Engineering, 8(2), 219–229.

[3] Munoz, S., Ortuno, M. T., Ramirez, J., & Yanez, J. (2005). Coloring fuzzy graphs. Omega, 33(3), 211–221.

[4] Samanta, S., Pramanik, T., & Pal, M. (2016). Fuzzy colouring of fuzzy graphs. Afrika Matematika, 27(1–2), 37–50.

[5] Eslahchi, C., & Onagh, B. N. (2006). Vertex-strength of fuzzy graphs. International Journal of Mathematics and Mathematical Sciences, 2006(1), 43614.

[6] Karunambigai, M. G., & Mathew, J. A. (2024). The chromatic number of certain families of fuzzy graphs. Cuestiones Fisioterapia, 53(3), 5153–5268.

[7] Karunambigai, M. G., & Mathew, J. A. (2025). The chromatic number of fuzzy paths and its related graphs. International Journal of Environmental Science,11(18s), 1858–1869.

[8] Karunambigai, M. G., & Mathew, J. A. (2025). The chromatic number of fuzzy cycles and its related graphs. International Journal of Environmental Science,11(20s), 3697–3709.

[9] Karunambigai, M. G., & Mathew, J. A. (in press). The chromatic number of homogeneous fuzzy caterpillars and its related graphs. International Journal of Applied Mathematics.

[10] Houcque, D. (2005). Introduction to MATLAB for engineering students (p. 1). Northwestern University.

[11] Moler, C., & Little, J. (2020). A history of MATLAB. Proceedings of the ACM on Programming Languages, 4(HOPL), 1–83.

[12] Abdul Majid, M., Huneiti, Z. A., Balachandran, W., & Balarabe, Y. (2013). MATLAB as a teaching and learning tool for mathematics: A literature review. International Journal of Arts & Sciences, 6(1), 205–221.

[13] Mathew, S., Mordeson, J. N., & Malik, D. S. (2018). Fuzzy graph theory. Springer International Publishing.

[14] Pal, M., Samanta, S., & Ghorai, G. (2020). Fuzzy cut vertices and fuzzy trees. In Modern trends in fuzzy graph theory (pp. 115–124). Springer.

[15] Gong, Z., & Zhang, J. (2022). Chromatic number of fuzzy graphs: Operations, fuzzy graph coloring and applications. Axioms, 11(12), 697.